# Virtual corrections to the NLO splitting functions for Monte Carlo: non-singlet case

**Oleksandr Gituliar**

oleksandr.gituliar@ifj.edu.pl

in collaboration with

**S.Jadach, A.Kusina, and M.Skrzypek**

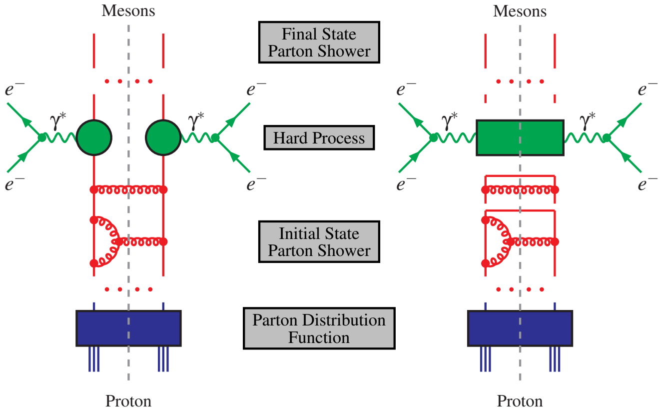
Institute of Nuclear Physics
Polish Academy of Sciences,
Cracow, Poland
http://www.ifj.edu.pl

"Matter to the deepest", Ustroń

6 September 2013

# Motivation: NLO Parton Shower for LHC (1/3)

Use **Collinear Factorization** theorem [R.K.Ellis et al. 1979] (works at inclusive level only!) to split **Parton Shower** into the product of **Splitting Functions**.




See S.Jadach's talk on how that works at exclusive level.

# Motivation: NLO Parton Shower for LHC (2/3)

Keep in mind that we consider:

- **axial gauge ($n^2 = 0$)**, since Factorization Theorem is complicated in covariant gauges
- **massless QCD** (all masses are 0)
- **collinear expansion**

# Motivation: NLO Parton Shower for LHC (3/3)

NLO Parton Shower Monte-Carlo for QCD does not exist!

## Brief history of Monte-Carlo for QCD

- LO Hard Process + LO Parton Shower – Pythia, Herwig (1980s)
- NLO Hard Process + LO Parton Shower – MC@NLO, PowHEG (2000s)
- NLO Hard Process + NLO Parton Shower – **KrKMC** (ongoing)

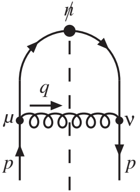
Aim of this work is to calculate **exclusive** NLO Splitting Functions suitable for Monte Carlo (KrKMC project – S.Jadach et al.)

# Definition of Splitting Functions

Splitting functions are defined in terms of projection operators and Feynman rules (axial gauge) in $m = 4 + 2\epsilon$ dimensions [Curci, Furmanski, Petronzio 1980]:

$$T(p, \{q_{\text{out}}, \dots\}, \epsilon) = \left[ \frac{\not{n}}{4\, p \cdot n} K \not{p} \right]$$

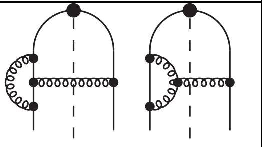
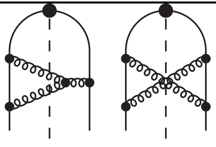
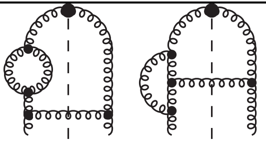
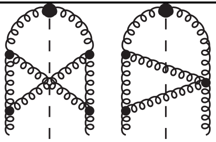
For example, LO Splitting Function is defined as




$$= \frac{g^2}{\mu^\epsilon} \mathrm{Tr} \left( \frac{\not{n}}{4\, p \cdot n} \frac{\not{p} - \not{q}}{(p-q)^2} \gamma^\mu \not{p} \gamma^\nu \frac{\not{p} - \not{q}}{(p-q)^2} \right) d_{\mu\nu}(q) \delta^+(q^2)$$

$$d_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu n_\nu + n_\mu q_\nu}{q \cdot n}$$

# Some examples of NLO Splitting Functions

| | Virtual | Real |
|---|---|---|
| Non-Singlet |  |  |
| Singlet |  |  |

# The Problem!

**Virtual** and **real** graphs **contain** higher-order **singularities**!
(which belong to 1- and 2-particle real phase space)

Impossible to construct NLO Parton Shower.

| | real | virtual |
|---|---|---|
| $1/\epsilon^3$ | -1 | 1 |
| $1/\epsilon^2$ | -1 | 2 |
| $\ln Q^2$ | -1 | 1 |

G.Heinrich, PhD thesis, 1998

| | real | virtual |
|---|---|---|
| $1/\epsilon^3$ | 0 | 0 |
| $1/\epsilon^2$ | 0 | 1 |
| $\ln Q^2$ | 0 | 0 |
| | S.Jadach et al. 2010 | this work |

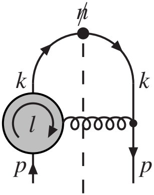
$Q$ – Hard Process scale

Monte Carlo needs a new calculation scheme with no higher-order singularities.

# Loop-momentum integration (1/3)

Definition of the splitting function leads to the expression

$T^{\mathrm{NLO}}_{\mathrm{virt}}(p, k, l, \epsilon) =$




$$= g^4 \left( C \underbrace{\frac{l \cdot a_i \ \cdots}{(l + a_j)^2 \ \cdots}}_{\text{Feynman}} \underbrace{\frac{1}{l \cdot n}}_{\text{axial}} + \ldots \right)$$

for example

$$g^4 \left( k \cdot p \frac{l \cdot k}{l^2 (l + p)^2} + k^2 \, k \cdot n \frac{l \cdot p \; l \cdot k}{l^2 (l + k)^2 (l + p)^2} \frac{1}{l \cdot n} + \ldots \right)$$

Loop-momentum integration is complicated because of axial gauge!

# Loop-momentum integration (2/3)

Source of **three** types of singularities

- Ultra-Violet (UV) $\leftarrow$ harmless (UV renormalization)
- Infra-Red (IR) $\leftarrow$ SOURCE OF THE PROBLEM
- Spurious $\leftarrow$ harmless, but BE CAREFUL

# Loop-momentum integration (3/3)

- **UV poles**
  - contribute to evolution of the coupling constant
  - subtracted by UV renormalization

$$\int \frac{d^m l}{(2\pi)^m} \frac{1}{l^2(l-p)^2} = i(4\pi)^{-2+\epsilon} \frac{\Gamma(1+\epsilon)}{\epsilon_{\rm uv}} \frac{B(1-\epsilon, 1-\epsilon)}{(p^2)^\epsilon}$$

- **IR poles**
  - define splitting functions
  - renormalized as stated in Collinear Factorization theorem
- **Spurious poles**
  - contribute to splitting functions
  - are non-physical and don't appear in the final results

# Regularization prescriptions

Dimensional Regularization

$$\frac{1}{x} \to \frac{1}{x^{1-\epsilon}}$$

leads to poles in $\epsilon_{\rm ir} \leftarrow$ DANGEROUS

$$\mathcal{I} = \int_0^1 \mathrm{d}x \, \frac{1}{x^{1-\epsilon}} = \frac{1}{\epsilon_{\rm ir}}$$

Principal Value Prescription

$$\frac{1}{x} \to \frac{x}{x^2+\delta^2}$$

leads to poles in $\delta \leftarrow$ HARMLESS

$$\mathcal{I} = \lim_{\delta\to 0} \int_0^1 \mathrm{d}x \frac{x}{x^2+\delta^2} = -\ln\delta$$

# Standard Approach

$$\mathcal{I}_3 = \int \frac{\mathrm{d}^m l}{(2\pi)^m} \frac{1}{l^2 (l-p)^2 (l-k)^2} = \int_0^1 \mathrm{d}z_1 \mathrm{d}z_2 \int \frac{\mathrm{d}^m l}{(2\pi)^m} \frac{1}{(l^2 + l \cdot A + B^2)^3}$$

$$k^2 \neq 0 \qquad p^2 = (p-k)^2 = 0$$

Use Principal Value prescription for axial denominators ONLY...

$$d^{\mu\nu} = g^{\mu\nu} - (l^\mu n^\nu + n^\mu l^\nu) \frac{1}{l_+} \quad \rightarrow \quad d^{\mu\nu}_{\mathrm{PV}} = g^{\mu\nu} - (l^\mu n^\nu + n^\mu l^\nu) \frac{l_+}{l_+^2 + \delta^2}$$

... and Dimensional regularization for all others!

$$\frac{1}{l_+} \rightarrow \frac{1}{l_+^{1-\epsilon}}$$

That leads to: $\mathcal{I}_3 = \frac{1}{\epsilon_{\mathrm{ir}}} \int_0^1 \mathrm{d}z_1 \mathrm{d}z_2 \cdots \simeq \frac{1}{\epsilon_{\mathrm{ir}}{}^2}$

# New Approach – The Solution of $1/\epsilon^3$ problem!

$$\mathcal{I}_3 = \int \frac{\mathrm{d}^m l}{(2\pi)^m} \frac{1}{l^2 (l-p)^2 (l-k)^2} = \int \mathrm{d}l_+ \int_0^1 \mathrm{d}z_1 \mathrm{d}z_2 \int \frac{\mathrm{d}l_- \mathrm{d}^{m-2} l_\perp}{(2\pi)^m} \frac{1}{(l^2 + l\cdot A + B^2)^3}$$

$$k^2 \neq 0 \qquad p^2 = (p-k)^2 = 0$$

**Use Principal Value prescription for all singularities in $l_+$!**

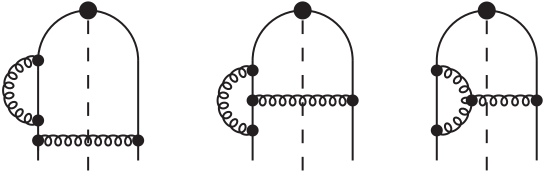
(not only for axial gauge denominators)

$$\mathcal{I}_3 = \frac{1}{\epsilon_{\mathrm{ir}}} \int \mathrm{d}l_+ \cdots \simeq \frac{\ln\delta}{\epsilon_{\mathrm{ir}}}, \quad \delta \to 0$$

# Properties of the Solution

- No higher-order poles $1/\epsilon^3$ and $1/\epsilon^2$ (for real case)
- No dependence on a hard process scale $Q$
- Total agreement with known calculations at inclusive level [Curci et al. 1980; Heinrich et al. 1998]
- NLO Splitting Functions are suitable for Monte Carlo Parton Shower

# Results: Non-singlet Splitting Functions



For each graph we calculated:

- UV counter-term
- **Inclusive** Splitting Function (for testing purposes)
- **Exclusive** Splitting Function (suitable for Monte Carlo!)

# **Axiloop** package (1/2)

https://github.com/gituliar/axiloop.git

**Fully automated tool for symbolic Splitting Functions calculation in axial gauge written in Mathematica.**

Features:

- Index contraction and trace calculation
- One-loop integration with various prescriptions in axial (and covariant gauges)
- Passarino-Veltman reduction of tensor integrals
- UV renormalization
- One-particle final state integration

# **Axiloop** package (2/2)

https://github.com/gituliar/axiloop.git

Design goals:

- Exclusive and Inclusive Splitting Functions at NLO
  - for singlet and non-singlet cases
  - for one- and two-particle final states
  - with geometrical cut-off for real emissions in 4 dimensions
- Corresponding hard processes at NLO
- All results in analytical form

# Summary

Done:

- **New regularization scheme** which eliminates higher-order singularities and reduces cancellations between real and virtual parts in Splitting Functions
- **Resulting SFs are suitable for Monte Carlo!**
- Complete calculation of non-singlet case (virtual + real)
- Fully automatic analytical calculations with **Axiloop** package (non-singlet virtual corrections)

In progress:

- singlet splitting functions

Possible future extension of **Axiloop**:

- two-loop virtual corrections
- real corrections