Higher-Order Corrections in QCD Evolution Equations and Tools for Their Calculation

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History of Monte-Carlo generators for LHC

- LO Hard Process + LO Parton Shower
  Pythia, Herwig (1980s)
- NLO Hard Process + LO Parton Shower
  MC@NLO, PowHEG (2000s)
- NLO Hard Process + NLO Parton Shower
  KrKMC (S.Jadach et al., ongoing)

For consistent simulations one needs PS one order less than Hard Process, for example NNLO(HP) + NLO(PS)

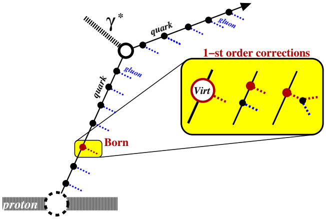
Motivation

We want/need to predict collider physics with better precision
and therefore
Build next-to-leading-order Parton Shower Monte-Carlo generator

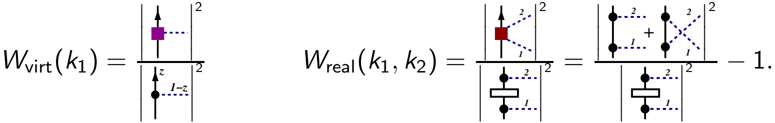
Building blocks for NLO PS Monte-Carlo:

- NLO Parton Shower algorithm
  [Jadach et al., Phys.Rev. D87, 2013]
- Real NLO Splitting Functions
  [Jadach et al. JHEP 1108, 2011]
- **Virtual NLO Splitting Functions**
  [my PhD thesis; O.Gituliar et al. Phys.Lett. B732 (2014) 218]
- NLO matrix elements (coefficient functions)

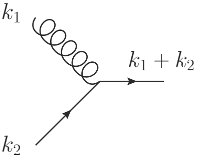
KrKMC – The Idea



The idea is to apply NLO corrections on top of the existing LO Parton Shower using corresponding weights with 1 and 2 real legs:



One-gluon emission in massless QCD



$$= \frac{1}{(k_1+k_2)^2} = \frac{1}{k_1^2 + k_2^2 + 2k_1 \cdot k_2} = \frac{1}{2E_1E_2(1-\cos\theta_{12})}$$

Where we took into account that

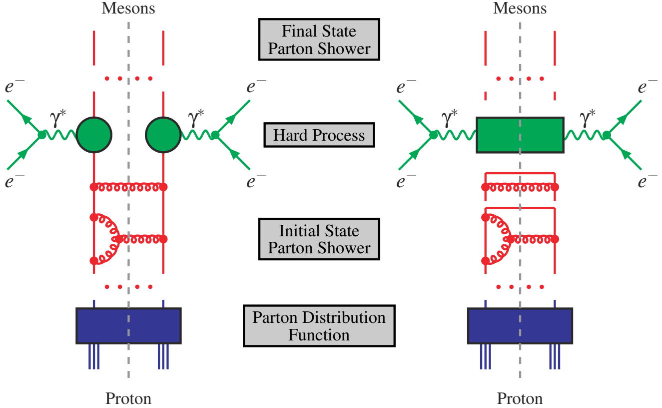
- for massless theory $m_i = 0$ and $E_i = |\vec{p}_i|$
- for observed particles $k_i^2 = m_i^2 \to 0$

In result we get ∞:

- for $E_i = 0$ (soft limit)
- for $\cos\theta_{12} = 1$ (collinear limit)

We should fix this issue since physical observable can not be ∞.

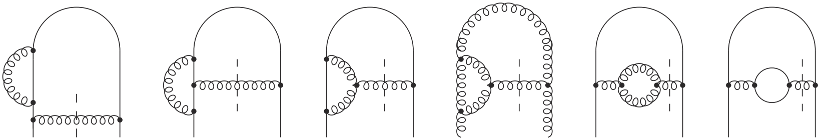
Generalized Ladder Expansion



In **axial gauges** collinear singularities come **only** from the integration over lines connecting 2PI kernels.

R. K. Ellis, H. Georgi, M. Machacek, H. D. Politzer, and G. G. Ross. *Nucl.Phys.* B152 (1979), p. 285

We have calculated **complete non-singlet** + extra



For all these graphs we have calculated:

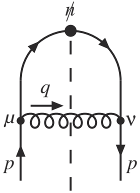
- **Exclusive SF** → suitable for **initial-state parton shower**
- **Ultra-violet counter-term** → **running** α_s
- **Inclusive SF** → cross-check with G. Curci, W. Furmanski, and R. Petronzio. *Nucl.Phys.* B175 (1980), p. 27; R. K. Ellis and W. Vogelsang. (1996). arXiv: hep-ph/9602356

Splitting Functions

Splitting Functions are build of

- projection operators for in- and outgoing lines
- Feynman rules (for light-cone gauge!)

$$W_n(p, q_1, q_2, \ldots, \epsilon, \delta) = \left[\frac{\not{n}}{4\, p \cdot n} K \not{p} \right]$$



$$= \frac{g^2}{\mu_f^\epsilon} \mathrm{Tr} \left(\frac{\not{n}}{4\, p \cdot n} \frac{\not{p} - \not{q}}{(p-q)^2} \gamma^\mu \not{p} \gamma^\nu \frac{\not{p} - \not{q}}{(p-q)^2} \right) d_{\mu\nu}(q) \delta^+(q^2)$$

$$d_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu n_\nu + n_\mu q_\nu}{q \cdot n}$$

Curci, Furmanski, and Petronzio,
"Evolution of parton densities beyond leading order: the nonsinglet case"

Algorithm for calculating Splitting Functions

1. Trace and index contraction in m dims
2. **Loop integration with appropriate regularization**
3. Renormalization (subtract UV poles)
4. Final-state integration in m dims

Dimensional Regularization & Principal Value

Dimensional Regularization

Transition to m-dimensional space-time is required

$$\int_0^1 \mathrm{d}l_+ \frac{1}{l_+} \to \int_0^1 \mathrm{d}^m l_+ \frac{1}{l_+} \overset{m\to 1+\epsilon}{\equiv} \int_0^1 \mathrm{d}l_+ \frac{1}{l_+^{1-\epsilon}} = \frac{1}{\epsilon}$$

Principal Value Prescription

Lives in 4 dims which makes it perfect for Monte-Carlo

$$\int_0^1 \mathrm{d}l_+ \frac{1}{l_+} \to \int_0^1 \mathrm{d}l_+ \left(\frac{1}{l_+}\right)_{\mathrm{PV}} \equiv \int_0^1 \mathrm{d}l_+ \frac{l_+}{l_+^2 + \delta^2} = -\ln\delta$$

Loop Integration

Calculation of the virtual corrections to NLO SFs requires evaluation of the following loop integrals:

$$\int \frac{\mathrm{d}^m l}{(2\pi)^m} \frac{\{1, l^\mu, l^\mu l^\nu\}}{l^2 (l+p_1)^2 (l+p_2)^2} \frac{1}{l\cdot n}, \qquad l_+ \equiv l\cdot n$$

where

- $m = 4 + 2\epsilon$
- n is constant gauge-fixing 4-vector, $n^2 = 0$
- p_1, p_2 are final-state 4-momenta, i.e. $p_1^2 = p_2^2 = 0$

Feynman denominators lead to IR and UV singularities which appear in the final result as $1/\epsilon$ terms.

Axial denominator is a source of spurious (unphysical) singularities and thus needs a special regularization prescription. A common choice is Principal Value prescription.

Loop Integration (examples)

- **UV** poles

$$\int \frac{d^m l}{(2\pi)^m} \frac{1}{l^2 (l+p)^2} \sim \frac{(4\pi)^{-2+\epsilon}}{(p^2)^\epsilon} \Gamma(1+\epsilon) \left(\frac{1}{\epsilon_{\mathrm{uv}}} + 2 \right) \qquad m = 4 - 2\epsilon$$

- **IR** & **Spurious** poles

$$\int \frac{d^m l}{(2\pi)^m} \frac{1}{l^2 (l+k)^2 (l+p)^2} \sim \frac{(4\pi)^{-2+\epsilon}}{(k^2)^{1+\epsilon}} \Gamma(1+\epsilon) \left(\frac{2 I_0 + \ln(1-x)}{\epsilon_{\mathrm{ir}}} + O(\epsilon^0) \right)$$

- **UV** & **IR** & **Spurious** poles appear in vector and tensor integrals

$$p^2 = (p-k)^2 = 0 \qquad k^2 \neq 0$$

$$I_0 = \int_0^1 \mathrm{d}x \frac{x}{x^2 + \delta^2} = -\ln\delta\,, \qquad \delta \to 0$$

New Principal Value

Standard Principal Value

Use PV for Axial and DR for Feynman denominators

$$\int d^m l \frac{1}{l^2(l+k)^2(l+p)^2}\frac{1}{l_+} = \int_0^1 dz_1 dz_2 \int dl_+ dl_- d^{m-2} l_\perp \frac{1}{(l^2 + l\cdot A + B^2)^3}\left(\frac{1}{l_+}\right)_{PV}$$

New Principal Value

Use PV for l_+ and DR for the remaining variables

$$\int d^m l \frac{1}{l^2(l+k)^2(l+p)^2}\frac{1}{l_+} = \int_0^1 dz_1 dz_2 \int (dl_+)_{PV} dl_- d^{m-2} l_\perp \frac{1}{(l^2 + l\cdot A + B^2)^3}\frac{1}{l_+}$$

in particular use

$$(dl_+)_{PV}\ \frac{1}{l_+^{1-\epsilon}} \to dl_+\, l_+^{\epsilon}\left(\frac{1}{l_+}\right)_{PV} = dl_+ \frac{l_+}{l_+^2+\delta^2}\left(1 + \epsilon \ln l_+ + \dots\right)$$

Example: three-point scalar Feynman integral

$$\int \frac{\mathrm{d}^m l}{(2\pi)^m} \frac{1}{l^2(l+k)^2(l+p)^2}$$

PV approach

$$= \frac{i}{(4\pi)^2|k^2|}\left(\frac{4\pi}{|k^2|}\right)^{\epsilon}\Gamma(1+\epsilon)\left(-\frac{1}{\epsilon_{\mathrm{ir}}^2}+\frac{\pi^2}{6}\right)$$

NPV approach

$$= \frac{i}{(4\pi)^2|k^2|}\left(\frac{4\pi}{|k^2|}\right)^{\epsilon}\Gamma(1+\epsilon)\Bigg(\frac{2I_0+\ln(1-x)}{\epsilon_{\mathrm{ir}}} - 4I_1 + 2I_0\ln(1-x) + \frac{\ln^2(1-x)}{2}\Bigg)$$

Axiloop: Features

Fully automated tool for symbolic calculation of Splitting Functions in light-cone gauge written in Mathematica.

- Vector algebra and trace calculation in n dims
- One-loop integration with various prescriptions in light-cone and covariant gauges
- Passarino-Veltman reduction of tensor integrals with separated IR and UV poles
- One-leg final-state integration

IntegrateLoop #1

```
In[1]:= IntegrateLoop[ 1/(l.l (l+p).(l+p)), l,
          Prescription -> "NPV"]

Out[1]= {
  {collected, $$[{}, {0, p}, {}]},
  {simplified, $$[{}, {0, p}, {}]},
  {integrated, {{short, Qv[p] T0[euv]},
                   Qv[p]
  {long, 2 Qv[p] - -----}}}}
                    euv
```

IntegrateLoop #2

```
In[1]:= $Get[
  IntegrateLoop[ l.k/(l.l (l+k).(l+k) (l+p).(l+p)), l,
    Prescription -> "NPV",
    SimplifyNumeratorAndDenominator -> False]
  ,
  {"integrated", "short"}
]


        Qv[k] (2 R2[eir] k.k + R1[eir] (k.k + p.p - q.q))
Out[2]= -------------------------------------------------
                              2 k.k
```

IntegrateLoop #3

```
In[1]:= $Get[
  IntegrateLoop[ l.k/(l.l (l+k).(l+k) (l+p).(l+p)), l,
    Prescription -> "NPV",
    SimplifyNumeratorAndDenominator -> True]
  ,
  {"integrated", "short"}
]
```

```
          Qv[k] R0[eir]   Qv[p] T0[euv]   Qv[q] T0[euv]
Out[1]= - ------------- + ------------- - -------------
                2               2               2
```

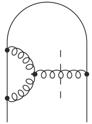
IntegrateLeg

```
In[1]:= WrLOqq = 2 g^2 (1+x^2 + (1-x)^2 eps)/((1-x) k.k);

In[2]:= IntegrateLeg[ WrLOqq, k ]

             2    2 eps        2             2
        -2 g   (Q )     Qr (1+x  + eps (1-x) ) (1 + eps Log[1-x])
Out[2]= -------------------------------------------------------
                            eps (1-x)
```

Graph (d) in NPV scheme

The **ultra-violet counter-term**

$$W_Z \;=\; \alpha_s^2\, \mathcal{C}_{qq}^{(d)}\, \frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\, \frac{1}{|k^2|}\, \frac{1}{\epsilon_{\rm uv}}\, \left(6 - 4\ln x - 8\ln(1-x) - 16 I_0\right) P_{qq}$$



The **exclusive (renormalized) splitting function**

$$W_R = \alpha_s^2\, \mathcal{C}_{qq}^{(d)}\, \frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\, \frac{1}{|k^2|} \left\{ \frac{1}{\epsilon}(6 - 4\ln x - 8\ln(1-x) - 16 I_0) \left(\left(\frac{|k^2|}{\mu_r^2} \right)^\epsilon - 1 \right) P_{qq} \right.$$

$$\left. + \left(p_{qq} \left(-14 + 16\mathrm{Li}_2(1) + 4\ln^2 x - 4\mathrm{Li}_2(1-x) + 8 I_0 (\ln x + \ln(1-x)) - 24 I_1 \right) - 2x \right) \left(\frac{|k^2|}{\mu_r^2} \right)^\epsilon \right\}$$

- W_R is finite as $\epsilon \to 0$ and can be used in 4 dims as in Monte-Carlo
- In PV approach W_R contains $1/\epsilon^2$ which remains as $\epsilon \to 0$, (though cancels in the end with $1/\epsilon^2$ in the real graph) [G.Heinrich, PhD thesis, 1998, Table 3.10]
- Divergent I_0 and I_1 are $\sim \ln\delta$ $(\delta \to 0)$, but live in 4 dims

Inclusive real+virt results in NPV

			SUM			SUM				SUM			SUM
	$(d): {}^1\!/_2\, C_F C_A$			$(c): C_F^2 - {}^1\!/_2\, C_F C_A$			$(e): C_F^2$	$(f): {}^1\!/_2\, C_F C_A$			$(g): C_F T_F$		
Double poles													
p_{qq}	−6	0	−6	−6	0	−6	6	44/3	−22/3	22/3	−8/3	4/3	−4/3
$p_{qq}\ \ln x$	4	0	4	4	0	4	−8	0	0	0	0	0	0
$p_{qq}\ \ln(1-x)$	8	0	8	0	0	0	0	−16	8	−8	0	0	0
$p_{qq}\ I_0$	16	0	16	8	0	8	−8	−16	8	−8	0	0	0
Single poles													
p_{qq}	−7	−4	−11	−7	0	−7	7	0	103/9	103/9	0	−10/9	−10/9
$p_{qq}\ \ln x$	0	−3/2	−3/2	0	−3/2	−3/2	0	0	11/3	11/3	0	−2/3	−2/3
$p_{qq}\ \ln(1-x)$	−3	8	5	−3	0	−3	3	22/3	−34/3	−4	−4/3	4/3	0
$p_{qq}\ \ln^2 x$	2	−1	1	2	−1	1	−2	0	0	0	0	0	0
$p_{qq}\ \ln x \ln(1-x)$	2	4	6	2	0	2	−4	0	−4	−4	0	0	0
$p_{qq}\ \ln^2(1-x)$	4	−2	2	0	0	0	0	−8	6	−2	0	0	0
$p_{qq}\ \mathrm{Li}_2(1)$	8	−2	6	4	0	4	−4	0	−4	−4	0	0	0
$p_{qq}\ \mathrm{Li}_2(1-x)$	−2	2	0	2	−2	0	0	0	0	0	0	0	0
$1-x$	−5/2	3/2	−1	−7/2	−15/2	−11	3	22/3	−4	10/3	−4/3	0	−4/3
$(1-x)\ \ln x$	2	0	2	2	0	2	−4	0	0	0	0	0	0
$(1-x)\ \ln(1-x)$	4	0	4	0	0	0	0	−8	4	−4	0	0	0
$1+x$	−1/2	1/2	0	1/2	−1/2	0	0	0	0	0	0	0	0
$(1+x)\ \ln x$	0	1/2	1/2	0	−7/2	−7/2	0	0	0	0	0	0	0
Spurious poles													
$p_{qq}\ I_0$	0	8	8	0	0	0	0	0	−4	−4	0	0	0
$p_{qq}\ I_0 \ln x$	4	4	8	4	0	4	−4	0	−4	−4	0	0	0
$p_{qq}\ I_0 \ln(1-x)$	12	−4	8	4	0	4	−4	−8	4	−4	0	0	0
$p_{qq}\ I_1$	−12	4	−8	−4	0	−4	4	0	4	4	0	0	0
$(1-x)\ I_0$	8	0	8	4	0	4	−4	−8	4	−4	0	0	0

Table 4.9: Contributions to the inclusive splitting function P_{qq} from the real and virtual topologies in the NPV prescription.

Summary

Done:

- **New Principal Value prescription** for calculating virt & real contributions to NLO Splitting Functions suitable for Monte-Carlo simulations in 4 dimensions
- **Complete results** for the non-singlet virt & real contributions
- **Axiloop package** for analytical calculation of virt contributions

In progress:

- Remaining virtual NLO splitting functions

Future:

- Two-loop virtual corrections
- Coefficient functions (hard process)
- Real NLO splitting functions in `Axiloop` package